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COUPLING EFFECTS IN THE ENTRAINMENT OF LOW-FREQUENCY FLUCTUATIONS

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We experimentally and numerically study the control of the low-frequency fluctuations exhibited by two mutually coupled semiconductor lasers. We observe how coupling can increase the response of an external weak periodic current, obtaining an entrainment of the power dropouts at a driving frequency. We compare the quality of the entrainment with the one obtained by a single semiconductor laser with optical feedback, showing the advantages of using the entrainment with coupling. We also present numerical results obtained from an extension of the Lang-Kobayashi model to a system of two mutually coupled semiconductor lasers.

Keywords: low-frequency fluctuations, entrainment by modulation, mutually coupled lasers, anticipative entrainment.

Semiconductor lasers exhibit a rich variety of dynamics when they are submitted to external reflections, as it could be expected in a typical time-delayed system. Although sometimes these reflections are undesirable, they can also be introduce in order to make the laser to operate at certain dynamical regimes. Among the different regimes of operation of a semiconductor laser with optical feedback, coherence collapse (CC) [1, 2] and low-frequency fluctuations (LFF) [3] have specially attracted the interest of different researchers. The LFF regime is achieved when the semiconductor laser is pumped near its threshold with moderate to low feedback levels. Under these conditions, the laser exhibits trains of fast pulses ($\sim$ 100 ps), combined with intensity dropouts at frequencies much lower than the typical of the system. These intensity dropouts, that occur at irregular times, are the well-known LFF's. Although its mechanism remains unclear, Sano [4], based in numerically simulations of the Lang-Kobayashi (LK) equations [5], interpreted the LFF as a crisis between the trajectory of the system with unstable saddle points. Although noise is presented in the system as the unavoidable spontaneous emission noise, its effect has been proved not to be relevant in the LFF appearance [6]. Beside the different work focused in the LFF explanation, some results have been obtained in order to stabilize [7,8] or control [9,10] the LFF appearance by different techniques. In the present paper we experimentally and numerically study the entrainment of the LFF by an external periodic forcing. We take advantage of the fact that coupled systems can enhance the response to an external driving [11], and compare the entrainment of a single semiconductor laser with optical feedback that is modulated by the pumping current [9], with a system of two mutually coupled semiconductor lasers where just one of the lasers is modulated. In the latter system, no external mirror is needed in order to observe the LFF [12,13]. We observe that the entrainment of the LFF is more effective in the coupled system. In addition to this, and due to the fact that less pumping current is needed, the external modulation is smoothed in the coupled system, observing solely a LFF entrainment in the laser output.

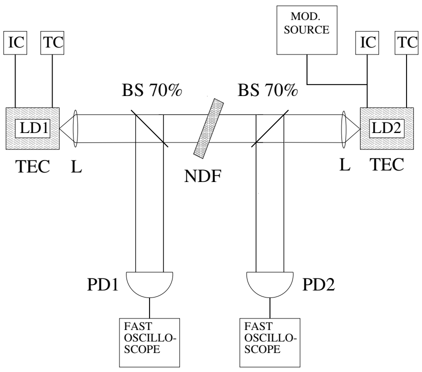

Fig 1. Experimental setup: LD, laser diode; BS, beam splitter; TEC, laser diode mount; PD, photodiode; IC, injection current source; TC, temperature controller; NDF, neutral density filter.

The experimental setup consists in two mutually coupled semiconductor lasers as shown in 1. The two low-power (~ 5 mW) ALGaInP semiconductor lasers emit at a wavelength of 650 nm. Both lasers are temperature controlled with an accuracy of $\pm 0.01\,^{\circ}$C and pumped with a low-noise source of ± 0.1 mA accuracy. Laser beams are collimated by two anti-reflection-coated laser objectives. A neutral density filter is placed between both lasers in order to control the coupling between both lasers. In the detection branch two fast photodetectors send the signal to a 500 MHz acquisition card. The distance between both lasers is 1.02 m, which correspond to a flying time of $\tau_c = 3.4$ ns. Solitary laser thresholds are $I_{th1} = 17.5$ mA and $I_{th2} = 17.3$ mA at the temperatures of $T_1 = 18.15\,^{\circ}$C and $T_2 = 22.25\,^{\circ}$C respectively. Finally we are able to modulate the pumping current of the laser with a sinusoidal wave introduced by an Agilent 33120 A function generator. With the aim of obtain the synchronization of both lasers we adjust the temperature and pumping current to $I_1 = 17.8$ mA, $I_2 = 17.7$ mA, $T_1 = 18.15\,^{\circ}$C and $T_2 = 22.25\,^{\circ}$C. At this values the detuning is low enough to achieve the synchronization in the LFF regime.

Although synchronization between two mutually coupled semiconductor lasers is observed in a range of detunings [10,12] without an influence in the dynamics of both lasers, the coupling time, τ_c, between both systems will appear in the synchronization. Thus, the intensity of the laser with higher frequency will always drop first than the intensity of the laser with lower frequency, which will fall just after the coupling time τ_c [10,12], exhibiting a leader-laggard role in the synchronization. Due to the detuning effects, we can make one laser to be the leader (laggard) in the dynamics by increasing (decreasing) its wavelength. From the starting point of both lasers synchronized in the LFF regime, with a low detuning in order to force one laser to be the leader in the dynamics, we introduce a $100MHz$ sinusoidal driving through the pumping current of the leader laser. In Fig.1 the amplitude of the periodic driving is increased, observing how the LFF period pass from irregular values to multiples of the modulation period and finally to follow the sinusoidal period when the modulation amplitude is high enough. It can be observed, the entrainment of the LFF in both lasers, although the laggard laser falls after the coupling time τ_c. Observing Fig.1 in detail, it can be seen a slight modulation in the output intensity of the leader (directly modulated laser) but not in the laggard, due to the fact that the laggard is synchronized with the nonlinear dynamics of the leader and not with the modulation. This phenomenon is used in encoded data transmission to recover a message [14].

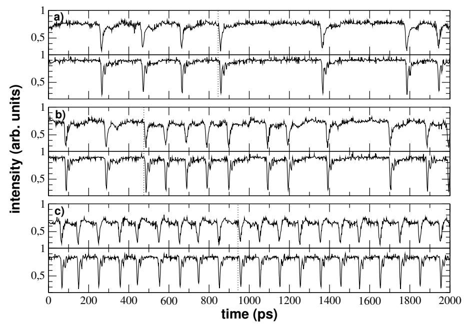

Fig 2. Time evolution of the intensity dropouts, for increasing values of modulation amplitude: 0.125 mA (a), 0.250 mA (b) and 0.500 mA (c). The modulated laser is in the upper figures. Note the filtering properties of the system in the laggard laser (lower figures), where modulation is smoothed.

In Fig.2 we compare the entrainment of the LFF obtained in a two mutually coupled semiconductor lasers (b) with the one obtained replacing one of the lasers by an external mirror (a) as in ref. [9]. In the coupled system, the response to the external forcing is enhanced by the coupling effects [11]. In this way, less modulation is needed making the entrainment more effective. In addition to this, the modulation signal, which is clearly observed in the output intensity of the single system, is smoothed in the coupled system, leading to a clean entrainment of the LFF. In this particular example, modulation current is 17.1% of the threshold current in the single laser with optical feedback (a) and 2.8% in the mutually coupled lasers (b). Note that Fig. 1-(b) is the laggard output intensity which is not directly modulated (current modulation is only present in the leader laser).

In order to numerically reproduce the experimental results we have developed

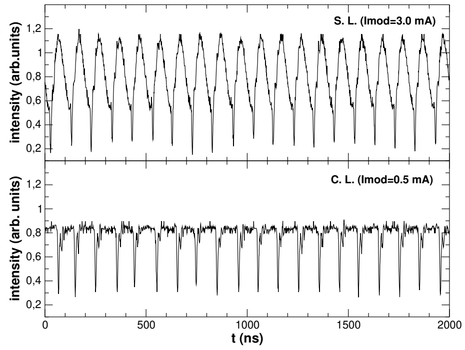

Fig 3. Intensity time trace (a) of a single laser (S.L.) with optical feedback and intensity time trace (b) of the laggard laser in two mutually coupled lasers (C.L.). Both systems are in the entrained LFF regime.

an extension of the L-K model for a two mutually coupled semiconductor lasers. The model describes the evolution of the envelope of the complex electric fields $E_{1,2}$ and the carrier numbers $N_{1,2}$ [6] by a set of rate equations, where the subindex 1 and 2 differences each laser:

$$\begin{aligned} \frac{dE_{1,2}}{dt} &= \frac{(1+i\alpha)}{2}\left[G_{1,2}-\gamma\right]E_{1,2} \pm i\Delta\omega E_{1,2} \\ &\qquad + \kappa e^{-i\Omega\tau_c} E_{2,1}(t-\tau_c) \qquad (1) \\ \frac{dN_{1,2}}{dt} &= \frac{I_{1,2}}{e} - \gamma_{e1,e2} N_{1,2} - G_{1,2}P_{1,2}(t) \qquad (2) \end{aligned}$$

where $P_{1,2}(t) = |E_{1,2}(t)|^2$ is the number of photons (optical intensity). The typical laser parameters that appear in the equations are the linewidth enhancement factor α (same value for both lasers) and the cavity losses γ. The last term of eq.(1) correspond to the coupling term, where κ is the coupling coefficient, τ_c is the time that takes to the light to go from one laser to the other and $\Omega = (\omega_1 + \omega_2)/2$ is the reference frequency, being ω_1 and ω_2 the corresponding free-running optical frequency of each laser. The gain coefficient $G_{1,2}$ is calculated by the following expression:

$$G_{1,2}(t) = \frac{[g(N_{1,2} - N_0)]}{[1 + sP_{1,2}(t)]} \qquad (3)$$

where g is the differential gain coefficient, N_0 is the carrier number at transparency and s is the gain saturation factor. The values used in the simulations are the following: $\alpha = 3.5$, $\kappa = 20$ ns^{-1}, $\tau_c = 3.4ns$, $g = 1.2 \times 10^{-8}$ ps^{-1}, $s = 5 \times 10^{-7}$ and $N_0 = 1.25 \times 10^8$. We have considered different values for the cavity losses and the cavity decay rate to avoid having the same lasers in order to obtain the same threshold currents as the observed in the experiments ($I_{th1} = 17.3$ mA and $I_{th2} = 17.5$ mA), thus $\gamma_1 = 0.687$ ps^{-1}, $\gamma_2 = 0.496$ ps^{-1}, $\gamma_{e1} = 0.601$ ns^{-1} and

$\gamma_{e2} = 0.651$ ns^{-1}. We introduce a slight detuning $\Delta\omega = (\omega_1 - \omega_2)/2 = 2$ GHz to force laser 1 to be the leader in the dropout dynamics due to its higher frequency. The external modulated signal is introduced in the system through the pumping current of laser 1 as $I_1 = I_{b1} + A_m \sin(2\pi t/T_m)$, where $I_{b1} = I_{th1}$ is the constant term of the pumping current. $A_m = 0.3$ and $T_m = 100$ ns are the amplitude and period of the sinus modulated signal. Laser 2 is pumped without external modulated with a pumping value of $I_2 = I_{th2}$.

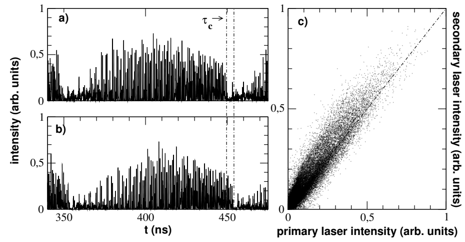

Fig 4. Numerical results corresponding to a mutually coupled lasers with an entrainment of the LFF. Note that intensity has not been filtered in order to observe the fast intensity pulses of both lasers. The delay time τ_c between the leader and the laggard is marked in the figure.

The numerical results obtained from the L-K extension perfectly match with the experimental observation, showing an entrainment of the LFF when modulation current is enough increased. Fig. 4 we represent the output intensity of both lasers when the entrainment is achieve. Numerical simulations allow us to observe the fast pulses of the lasers in the LFF regime, which were not observable in the experiments due to the fast dynamics involved. It can be seen how the pulses disappear to produce an intensity dropout and how the synchronization of the laggard is delayed a time τ_c respect to the laggard.

In order to quantify the amount of coupling needed to observe the entrainment we have studied the response of the system adjusting the coupling with a neutral density filter. From the system perfectly entrained at the maximum transparency, that is, the maximum amount of light that can be interchanged, we have reduced the coupling between both lasers. The experimental results are presented in Fig. 5, where it can be seen that also for intermediate values of coupling, the entrainment of the system still exists, only when the coupling is reduce more than the 50% of the maximum value, the quality of the entrainment is degradated.

The numerical results corresponding to the coupling efficiency are presented in Fig. 6, where we can also reproduce the experimental observation by the L-K extension. Note that in these numerical time series, the output intensity has been filtered with a $500MHz$ Butterworth filter with the aim of obtain the same signal as the photodetector.

In conclusion, the entrainment of two mutually coupled semiconductor lasers has been analyzed. We have focused the study in the effects of the coupling, concluding that coupling helps the system to response to an harmonic external signal. We observe advantages in using a coupled system in front of a single one, from the point of view of the quality and efficiency of the entrainment. Numerical simulations obtained from an extension of the L-K model recover the observed experimental

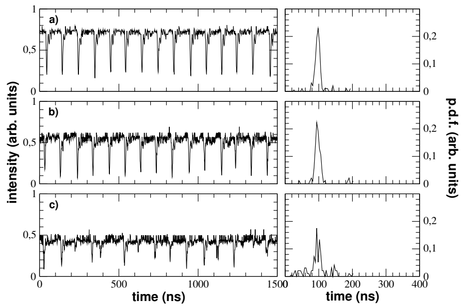

Fig 5. Experimental time series and p.d.f. of the laggard laser when the coupling between leader-laggard is decreased.In relation with the maximum coupling: a) 100%, b) 83.9% and c) 45.8%.

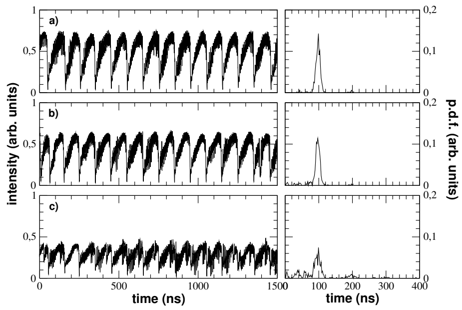

Fig 6. Numerical results corresponding to the experimental observations presented in Fig. 4.

results, from the point of view of the synchronization and the coupling efficiency.

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